

Twist-3 effects for polarized virtual photon structure function g_2^γ

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We investigate twist-3 effects in the polarized virtual photon. The structure function g_2^γ , which exists only for the virtual photon target and can be measured in future polarized e^+e^- collider experiments, receives both twist-2 and twist-3 contributions. The twist-3 part is analyzed in pure QED interaction as well as in LO QCD. We find the twist-3 contribution is appreciable for the photon in contrast to the nucleon case.

1. Introduction

In experiments of polarized deep inelastic lepton-nucleon scattering, we can obtain information on the two spin-dependent structure functions g_1^{nucl} and g_2^{nucl} of the nucleon. In the language of operator product expansion (OPE), the twist-2 operators contribute to g_1^{nucl} in the leading order of $1/Q^2$. On the other hand, g_2^{nucl} receives contributions from both twist-2 and twist-3 operators in the leading order. The twist-2 part of g_2^{nucl} can be extracted, once g_1^{nucl} is measured, by so-called Wandzura-Wilczek (WW) relation [1]:

$$g_{2,tw.2}^{\text{nucl}}(x, Q^2) = g_2^{\text{nucl}, WW}(x, Q^2) \\ \equiv -g_1^{\text{nucl}}(x, Q^2) + \int_x^1 \frac{dy}{y} g_1^{\text{nucl}}(y, Q^2). \quad (1)$$

The difference, $\bar{g}_2^{\text{nucl}} = g_2^{\text{nucl}} - g_2^{\text{nucl}, WW}$, contains the twist-3 contribution. The experimental data so far obtained show that the twist-3 contribution to g_2^{nucl} appears to be negligibly small [2,3].

In recent years, there has been growing interest in the study of spin structures of photon. The polarized photon structure functions can be measured by the polarized e^+e^- collision experiments in the future linear colliders (Fig.1), where $-Q^2$ ($-P^2$) is the mass squared of the probe (target) photon. For the virtual photon target, there appears two structure functions $g_1^\gamma(x, Q^2, P^2)$ and $g_2^\gamma(x, Q^2, P^2)$, which are the analogues to the

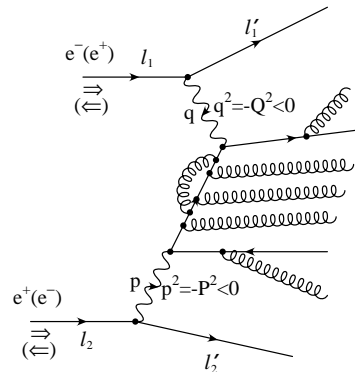


Figure 1. Deep inelastic scattering on a polarized virtual photon in polarized e^+e^- collision, $e^+e^- \rightarrow \text{hadrons}$ (quarks and gluons). The mass squared of the probe (target) photon is $-Q^2$ ($-P^2$) ($P^2 \ll Q^2$).

spin-dependent nucleon structure functions g_1^{nucl} and g_2^{nucl} , respectively.

Now we may ask about the photon structure function g_2^γ : (i) Does g_2^γ also receive twist-3 contribution? (ii) If so, is it small like the nucleon case, or, appreciable? (iii) Does the WW relation also hold for g_2^γ , in other words, is the twist-2 part of g_2^γ expressible in terms of g_1^γ ? (iv) Does any complication occur in the QCD analysis for g_2^γ ? These issues will be discussed [4] in the following.

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2. g_2^γ in Parton Model

Let us begin with the analysis of g_1^γ and g_2^γ in the simple parton model, in the kinematical region $P^2 \ll Q^2$. Evaluating the box diagrams (massless quark-loops) depicted in Fig.2 with the power corrections of P^2/Q^2 being neglected, we obtain

$$g_1^{\gamma(\text{Box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[(2x-1) \ln \frac{Q^2}{P^2} - 2(2x-1)(\ln x + 1) \right], \quad (2)$$

$$g_2^{\gamma(\text{Box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[-(2x-1) \ln \frac{Q^2}{P^2} + 2(2x-1) \ln x + 6x - 4 \right], \quad (3)$$

where $x = Q^2/(2p \cdot q)$, $\langle e^4 \rangle = \sum_{i=1}^{N_f} e_i^4/N_f$ with N_f being the number of active quark flavours and $\alpha = e^2/4\pi$.

First, note that $g_2^{\gamma(\text{Box})}$ satisfies the Burkhardt-Cottingham (BC) sum rule [5],

$$\int_0^1 dx g_2^{\gamma(\text{Box})}(x, Q^2, P^2) = 0. \quad (4)$$

In fact, we will see from the OPE analysis in Sec.3 that the BC sum rule for g_2^γ generally holds in the deep-inelastic region $Q^2 \gg P^2$. Now we apply the WW relation to the above results for $g_1^{\gamma(\text{Box})}$ and $g_2^{\gamma(\text{Box})}$, and define

$$g_2^{\gamma\text{WW}(\text{Box})}(x, Q^2, P^2) \equiv -g_1^{\gamma(\text{Box})}(x, Q^2, P^2) + \int_x^1 \frac{dy}{y} g_1^{\gamma(\text{Box})}(y, Q^2, P^2). \quad (5)$$

The difference, $\bar{g}_2^{\gamma(\text{Box})} = g_2^{\gamma(\text{Box})} - g_2^{\gamma\text{WW}(\text{Box})}$, is then given by

$$\bar{g}_2^{\gamma(\text{Box})} = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[(2x-2-\ln x) \ln \frac{Q^2}{P^2} - 2(2x-1) \ln x + 2(x-1) + \ln^2 x \right], \quad (6)$$

and its n -th moment is

$$\bar{g}_{2,n}^{\gamma(\text{Box})} = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \frac{n-1}{n} \times \left[-\frac{1}{n(n+1)} \ln \frac{Q^2}{P^2} + \frac{2}{(n+1)^2} - \frac{2}{n^2} \right]. \quad (7)$$

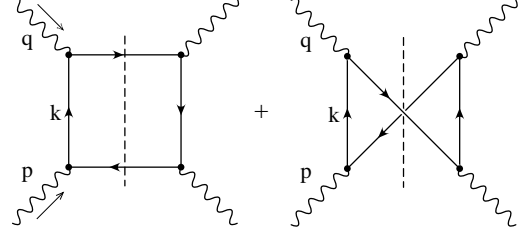


Figure 2. The box diagrams contributing to g_1^γ and g_2^γ in pure QED interaction.

In Fig.3, we plot the parton model results, $g_1^{\gamma(\text{Box})}$, $g_2^{\gamma(\text{Box})}$ and $\bar{g}_2^{\gamma(\text{Box})}$ given in Eqs.(2, 3, 6), as functions of x for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$. We can see that $\bar{g}_2^{\gamma(\text{Box})}$ is comparable in magnitude with $g_2^{\gamma(\text{Box})}$ for large region of x . It is now expected by analogy with the nucleon case that $\bar{g}_2^{\gamma(\text{Box})}$ arises from the twist-3 effects. In fact, we will be convinced in Sec. 3 that $\bar{g}_2^{\gamma(\text{Box})}$ is the twist-3 contribution.

3. OPE and Pure QED Effects on g_2^γ

Applying OPE for the product of two electromagnetic currents, we get for the μ - ν antisymmetric part,

$$\begin{aligned} & i \int d^4x e^{iq \cdot x} T(J_\mu(x) J_\nu(0))^A \\ &= -i \epsilon_{\mu\nu\lambda\sigma} q^\lambda \sum_{n=1,3,\dots} \left(\frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-1}} \quad (8) \\ & \times \left\{ \sum_i E_{(2)i}^n R_{(2)i}^{\sigma\mu_1 \cdots \mu_{n-1}} + \sum_i E_{(3)i}^n R_{(3)i}^{\sigma\mu_1 \cdots \mu_{n-1}} \right\}, \end{aligned}$$

where $R_{(2)i}^n$ and $R_{(3)i}^n$ are the twist-2 and twist-3 operators, respectively, and $E_{(2)i}^n$ and $E_{(3)i}^n$ are corresponding coefficient functions. The twist-2 operators $R_{(2)i}^n$ have totally symmetric Lorentz indices $\sigma\mu_1 \cdots \mu_{n-1}$, while the indices of twist-3 operators $R_{(3)i}^n$ are totally symmetric among $\mu_1 \cdots \mu_{n-1}$ but antisymmetric under $\sigma \leftrightarrow \mu_i$. Thus the ‘‘matrix elements’’ of operators $R_{(2)i}^n$

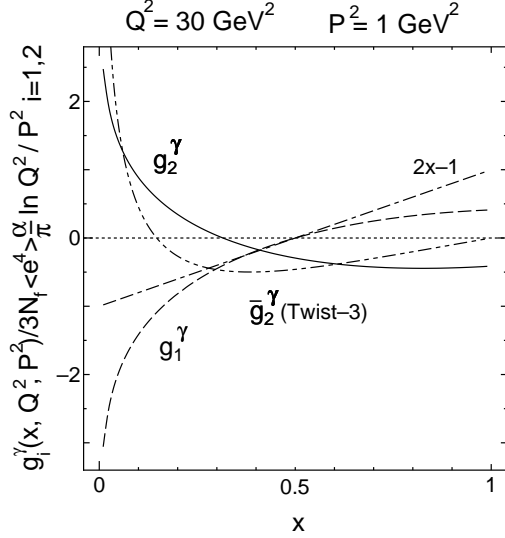


Figure 3. The Box-diagram contributions to g_1^γ (dashed line), g_2^γ (solid line) and \bar{g}_2^γ (dash-2dotted line) for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$. The $2x - 1$ line shows the leading logarithmic term of g_1^γ .

and $R_{(3)i}^n$ sandwiched by two photon states with momentum p have the following forms:

$$\begin{aligned} \langle 0 | T(A_\rho(-p) R_{(2)i}^{\sigma\mu_1 \dots \mu_{n-1}} A_\tau(p)) | 0 \rangle_{\text{Amp}} \\ = -i a_{(2)i}^n \epsilon_{\rho\tau\alpha} \{ \sigma p^{\mu_1} \dots p^{\mu_{n-1}} \} p^\alpha - (\text{traces}), \\ \langle 0 | T(A_\rho(-p) R_{(3)i}^{\sigma\mu_1 \dots \mu_{n-1}} A_\tau(p)) | 0 \rangle_{\text{Amp}} \\ = -i a_{(3)i}^n \epsilon_{\rho\tau\alpha} [\sigma, p^{\{\mu_1} \dots p^{\mu_{n-1}} \}] p^\alpha - (\text{traces}), \end{aligned}$$

where the suffix 'Amp' stands for the amputation of the external photon lines. Then the moment sum rules for g_1^γ and g_2^γ are written as follows:

$$\begin{aligned} \int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) &= \sum_i a_{(2)i}^n E_{(2)i}^n(Q^2), \\ \int_0^1 dx x^{n-1} g_2^\gamma(x, Q^2, P^2) &= \frac{n-1}{n} \\ &\times \left[- \sum_i a_{(2)i}^n E_{(2)i}^n(Q^2) + \sum_i a_{(3)i}^n E_{(3)i}^n(Q^2) \right]. \end{aligned}$$

From this general OPE analysis we conclude:

(i) The BC sum rule holds for g_2^γ ,

$$\int_0^1 dx g_2^\gamma(x, Q^2, P^2) = 0. \quad (9)$$

(ii) The twist-2 contribution to g_2^γ is expressed by the WW relation

$$\begin{aligned} -\frac{n-1}{n} \sum_i a_{(2)i}^n E_{(2)i}^n(Q^2) \\ = \int_0^1 dx x^{n-1} g_2^{\gamma\text{WW}}(x, Q^2, P^2), \quad (10) \end{aligned}$$

with

$$\begin{aligned} g_2^{\gamma\text{WW}}(x, Q^2, P^2) \\ \equiv -g_1^\gamma(x, Q^2, P^2) + \int_x^1 \frac{dy}{y} g_1^\gamma(y, Q^2, P^2). \quad (11) \end{aligned}$$

(iii) The difference, $\bar{g}_2^\gamma = g_2^\gamma - g_2^{\gamma\text{WW}}$, contains only the twist-3 contribution,

$$\begin{aligned} \int_0^1 dx x^{n-1} \bar{g}_2^\gamma(x, Q^2, P^2) \\ = \frac{n-1}{n} \left[\sum_i a_{(3)i}^n E_{(3)i}^n(Q^2) \right]. \quad (12) \end{aligned}$$

Let us now analyze the twist-3 part of g_2^γ in pure QED, i.e., switching off the quark-gluon coupling, in the framework of OPE and the renormalization group (RG) method. In this case the relevant twist-3 operators are the quark and photon operators, which are given, respectively, by

$$\begin{aligned} R_{(3)q}^{\sigma\mu_1 \dots \mu_{n-1}} \\ = i^{n-1} e_q^2 \bar{\psi} \gamma_5 \gamma^{[\sigma} D^{\{\mu_1} \dots D^{\mu_{n-1}} \} \psi, \quad (13) \end{aligned}$$

$$\begin{aligned} R_{(3)\gamma}^{\sigma\mu_1 \dots \mu_{n-1}} \\ = \frac{1}{4} i^{n-1} \epsilon^{[\sigma}{}_{\alpha\beta\gamma} F^{\alpha\{\mu_1} \partial^{\mu_2} \dots \partial^{\mu_{n-1}} \} F^{\beta\gamma]}, \quad (14) \end{aligned}$$

where e_q is the quark charge, $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative, $F^{\alpha\beta}$ is the photon field strength, $\{ \}$ means complete symmetrization over the indices, while $[\sigma, \mu_j]$ denotes antisymmetrization on $\sigma\mu_j$, and trace terms are omitted. With the choice of the above photon operator $R_{(3)\gamma}^n$, we have $a_{(3)\gamma}^n = 1$.

Solving the RG equation for the coefficient functions corresponding to operators $R_{(3)q}^n$ and

$R_{(3)\gamma}^n$, we obtain, to lowest order in α ,

$$\begin{aligned} E_{(3)q}^n\left(\frac{Q^2}{\mu^2}, \alpha\right) &= 1 + \mathcal{O}(\alpha) \\ E_{(3)\gamma}^n\left(\frac{Q^2}{\mu^2}, \alpha\right) &= \frac{\alpha}{8\pi} K_{(3)q}^n \ln \frac{Q^2}{\mu^2} + \frac{\alpha}{4\pi} 3e_q^4 B_{(3)\gamma}^n, \end{aligned} \quad (15)$$

where $K_{(3)q}^n$ is the mixing anomalous dimension between the twist-3 photon operator $R_{(3)\gamma}^n$ and quark operator $R_{(3)q}^n$ and is given by

$$K_{(3)q}^n = -24e_q^4 \frac{1}{n(n+1)}. \quad (16)$$

The “matrix element” $a_{(3)q}^n$ of the quark operator $R_{(3)q}^n$ between the photon states is calculated to be

$$a_{(3)q}^n = \frac{\alpha}{4\pi} \left(-\frac{1}{2} K_{(3)q}^n \ln \frac{P^2}{\mu^2} + 3e_q^4 A_{(3)q}^n \right). \quad (17)$$

Inserting Eqs.(15)-(17) into (12) and remembering $a_{(3)\gamma}^n = 1$, we obtain for the n -th moment of \bar{g}_2^γ in pure QED,

$$\begin{aligned} \bar{g}_2^{\gamma, n}|_{\text{QED}} &= \frac{n-1}{n} \frac{\alpha}{4\pi} 3e_q^4 \left\{ -\frac{4}{n(n+1)} \ln \frac{Q^2}{P^2} \right. \\ &\quad \left. + A_{(3)q}^n + B_{(3)\gamma}^n \right\}. \end{aligned} \quad (18)$$

The dependence on the renormalization point μ disappears. And we note that although $A_{(3)q}^n$ and $B_{(3)\gamma}^n$ are individually renormalization-scheme dependent, the sum $A_{(3)q}^n + B_{(3)\gamma}^n$ is not. The calculation of box diagrams in Fig.2 gives

$$A_{(3)q}^n + B_{(3)\gamma}^n = 8 \left\{ \frac{1}{(n+1)^2} - \frac{1}{n^2} \right\}. \quad (19)$$

Now adding all the quark contributions of active flavours and replacing $3e_q^4$ in (18) with $3N_f \langle e^4 \rangle$, we find that the result is nothing but $\bar{g}_{2, n}^{\gamma(\text{Box})}$ given in Eq.(7), which is derived from the box-diagram calculation. Thus it is now clear that $\bar{g}_{2, n}^{\gamma(\text{Box})}$ is indeed the twist-3 contribution.

4. QCD Effects on g_2^γ

We now switch on the quark-gluon coupling and consider the QCD effects on \bar{g}_2^γ , the twist-3 part of g_2^γ . In the nucleon case, the analysis of

\bar{g}_2^{nucl} , the twist-3 part of the structure function g_2^{nucl} , turns out to be very complicated [6]. This is due to the fact that the number of participating twist-3 operators grows with spin (moment of \bar{g}_2^{nucl}) and that these operators mix among themselves through renormalization. Therefore, the Q^2 evolution equation for the moments of \bar{g}_2^{nucl} cannot be written in a simple form, but in a sum of terms, the number of which increases with spin.

The same is true for \bar{g}_2^γ . However, in certain limits the analysis for the moments of \bar{g}_2^γ becomes tractable. One is when n is a small number and the other is the large N_C (the number of colours) limit for the analysis of $\bar{g}_2^{\gamma(NS)}$, the flavour non-singlet part of \bar{g}_2^γ . Indeed, for $n = 3$ (the non-trivial lowest moment), we can get all the information on the necessary anomalous dimensions of participating operators, and thus we obtain the LO QCD prediction for the third moment of \bar{g}_2^γ [4]. On the other hand, for large N_C , we can evade the problem of operator mixing for $\bar{g}_2^{\gamma(NS)}$, and obtain the moments of $\bar{g}_2^{\gamma(NS)}$ in a compact form for all n .

In the case of $\bar{g}_2^{\text{nucl}(NS)}$, the twist-3 and flavour nonsinglet part of the nucleon structure function g_2^{nucl} , it has been observed [7,8] that at large N_C , the operators involving gluon field strength $G_{\mu\nu}$ decouple from the evolution equation of $\bar{g}_2^{\text{nucl}(NS)}$, and the whole contribution in LO is represented by one type of operators. In the photon case, the relevant twist-3 operators for $\bar{g}_2^{\gamma(NS)}$ are

$$\begin{aligned} R_{(3)F}^{\sigma\mu_1 \dots \mu_{n-1}} \\ = i^{n-1} \bar{\psi} \gamma_5 \gamma^{[\sigma} D^{\{\mu_1] \dots D^{\mu_{n-1}\}} Q^{\text{ch}} \psi, \end{aligned} \quad (20)$$

and the photon operators $R_{(3)\gamma}^n$ given in Eq.(14). Here $D_\mu = \partial_\mu - igA_\mu^a T^a + ieA_\mu$ is the covariant derivative, and Q^{ch} is the quark-charge factor and defined by $Q^{\text{ch}} = Q^2 - \langle e^2 \rangle \mathbf{1}$, where Q is the $N_f \times N_f$ quark-charge matrix, $\langle e^2 \rangle = \sum_{i=1}^{N_f} e_i^2 / N_f$ and $\mathbf{1}$ is an $N_f \times N_f$ unit matrix. In the approximation of neglecting terms of order $\mathcal{O}(1/N_C^2)$ and thus putting $2C_F = C_G$, the mixing anomalous dimensions between $R_{(3)F}^n$ and other hadronic (quark and gluon) operators turn out to vanish. Those which remain non-zero are only the (F, F) element and the mixing anomalous dimension be-

tween $R_{(3)F}^n$ and the photon operator $R_{(3)\gamma}^n$:

$$\hat{\gamma}_{n,FF}^{(0)} = 8C_F(S_n - \frac{1}{4} - \frac{1}{2n}), \quad (21)$$

$$\text{with } S_n = \sum_{j=1}^n \frac{1}{j},$$

$$K_{n,F}^{(0)} = -24N_f(\langle e^4 \rangle - \langle e^2 \rangle^2) \frac{1}{n(n+1)}. \quad (22)$$

The corrections are of $\mathcal{O}(1/N_C^2)$, about 10% for QCD ($N_C = 3$). Using the above results, we find that, for large N_C , the n -th moment of $\bar{g}_2^{\gamma(NS)}$ in LO QCD is given by

$$\begin{aligned} & \int_0^1 dx x^{n-1} \bar{g}_2^{\gamma(NS)}(x, Q^2, P^2) \\ &= \frac{n-1}{n} \frac{\alpha}{4\pi} \frac{2\pi}{\beta_0 \alpha_s(Q^2)} K_{n,F}^{(0)} \frac{1}{1 + \hat{\gamma}_{n,FF}^{(0)}/2\beta_0} \\ & \times \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\hat{\gamma}_{n,FF}^{(0)}/2\beta_0 + 1} \right\}, \quad (23) \end{aligned}$$

where $\alpha_s(Q^2)$ is the QCD running coupling constant and $\beta_0 = (11N_C - 2N_f)/3$ is the one-loop coefficient of the β function.

We perform the Mellin transform of Eq.(23) to get $\bar{g}_2^{\gamma(NS)}(x, Q^2, P^2)$ as a function of x . The result is plotted in Fig.4. Comparing with the pure QED box-graph contribution, we find that the LO QCD effects are sizable and tend to suppress the structure function $\bar{g}_2^{\gamma(NS)}$ both in the large x and small x regions, so that the vanishing $n = 1$ moment of $\bar{g}_2^{\gamma(NS)}$, i.e. the BC sum rule, is preserved.

5. Conclusion

We have analyzed the twist-3 effects in g_2^γ for the virtual photon target, in pure QED interaction as well as in LO QCD. We have found that the twist-3 contribution is appreciable for the photon in contrast to the nucleon case. In this sense, the virtual photon structure function g_2^γ provides us with a good testing ground for studying the twist-3 effects. We expect that the future polarized version of e^+e^- colliders may bring us important information on spin structures of photon.

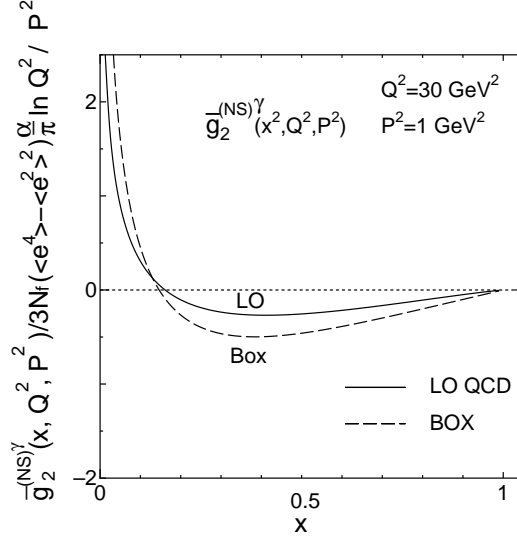


Figure 4. The Box-diagram (dashed line) and the QCD LO (solid line) contributions for large N_C to the flavour nonsinglet photon structure function $\bar{g}_2^{\gamma(NS)}(x, Q^2, P^2)$ for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$ for $N_f = 3$.

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